# Topography-Surveying

## Chapter 4

# Quality of Measurements

Institut des Sciences & Techniques Appliquées
University of Tlemcen
جامعة تلمسان

ISTA- University of Tlemcen

Dr. Abdennasser TACHEMA

# What Is It About?...

Topography requires the observation of a large number of measurements. Let us consider the distance between two points that are perfectly and very precisely defined: this distance is unique and has only one value, called the true value. This value is theoretical (ideal) and impossible to know exactly.If we measure this distance several times, we approach the true value, and the value that is retained—in this case, the arithmetic mean—is called the conventionally true value of the distance.

# DEFINITIONS [1]

## *Approximate Numbers*

An approximate number is any decimal representation that is not exact.

$\pi$ = 3,14159… , 2/3 = 0.666….

## *Significant Figures*

Significant figures are the digits that carry meaning, that is, those whose accuracy is known with certainty.

If a distance is given as 274.3 m, then: 274,3 m → 274,25<274,3<274,35

In this case, the number has four significant figures: three certain digits and the fourth being an approximation.

## *Rounded Numbers*

When a number contains digits beyond the achievable precision, or for obvious practical reasons, it may be necessary to retain only a specified number of significant figures from an approximate number.

| | | | | |
|---|---|---|---|---|
| Ex. | 832,4165 | → | 832,4 | rounding down |
| | 41,0047 | → | 41,00 | rounding down |
| | 55,0089 | → | 55,01 | rounding up |
| | 0,0046685 | → | 0,004669 | rounding up |

# DEFINITIONS [2]

***Nature of Measurements***

**Dependent Measurements**: When the operator is influenced by a previously known measurement.

**Conditioned Measurements**: When there exists a theoretical relationship between measurements (e.g., the sum of the angles of a polygon).

**Independent Measurements**: When the measurements are neither dependent nor conditioned.

**Repeated Measurements**: When measurements are taken again under the same conditions.

**Multiple Measurements**: Those taken cumulatively.

***Precision and Accuracy***

**Precision implies refinement in measurement and close agreement among repeated measurements.**

**Accuracy refers to the closeness of a measurement to its true value.**

# ERRORS [1]

In general, and particularly in surveying, physical measurements are subject to inaccuracies, which can be classified as **blunders** and **errors**.

**Blunders:** These are inaccuracies resulting from carelessness, oversight, or misunderstanding, such as reading 35 instead of 53. Blunders generally indicate incompetence and are usually detected during control measurements.

**Errors:** These are inaccuracies arising from the inevitable imperfections of instruments and human senses. Errors are generally small, but their accumulation can become significant.

**Types of Errors**

Error analysis shows that:

Some errors are **systematic** and unavoidable; in fact, they can often be identified and eliminated through proper measurement procedures or by evaluating them.

Others are **random** or **accidental**, arising purely by chance. They follow the laws of probability.

The surveyor's challenge, therefore, is to account for these errors in the final measured value and to define the acceptable limits for such errors.

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# ERRORS [2]

**Measurements Yield Non-Identical Values Because:**

1.Observations are made by humans, not robots; gestures and manipulations are therefore never exactly identical.

2.Observation conditions vary: temperature changes alter the length of a steel tape, differences in atmospheric pressure affect the waves of distance-measuring instruments, etc.

3.Instruments, no matter how precise, are of human design and manufacture and are inevitably subject to errors: axes of a theodolite may not be perfectly perpendicular, the optical axis of a level may not be perfectly horizontal, etc.

**True Errors and Apparent Errors**

Regardless of the source, an error is theoretically estimated as the difference between a measurement and the perfect value that should have been obtained—these are called **true errors**.

$$e = x - \mu$$

True errors are practically never known, as the perfect value is beyond the observer's reach. Therefore, the focus is on **apparent errors**, also called **residuals** or **probable deviations**, which can be estimated by the difference between each measurement and the mean of a set of similar measurements of the same object.

# TERMINOLOGY

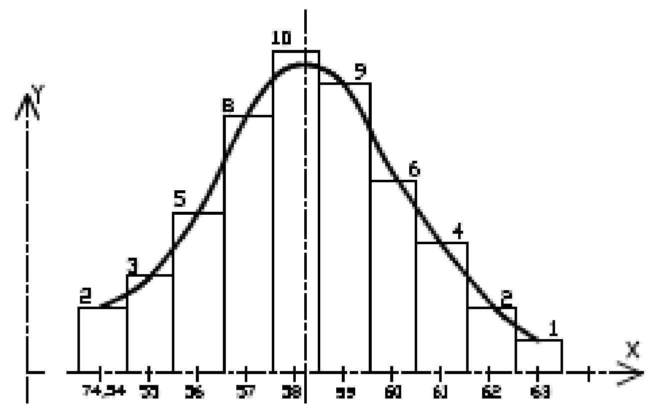
- **Measurement (Mesurage)**: The set of experimental operations aimed at determining the value of a quantity. In surveying, the term measurement is commonly used.
- **Measurement Method**: The mode of comparison used. There are two main types:
- Direct Measurement: Comparing the quantity directly with a standard.
- Indirect Measurement: Determining the quantity through calculations or relationships with other measured quantities.
- **True Value of a Quantity**: The value that characterizes a perfectly defined quantity. This is an ideal concept that is generally unknown.
- **Conventionally True Value of a Quantity**: An approximate value of the true value of a quantity, where the difference between the two can usually be neglected.
- **Measurement Error**: The discrepancy between the measurement result and the value, whether it is the true value, the conventionally true value, or the arithmetic mean of a series of measurements.
- **Blunders or Gross Errors**: Large, unacceptable errors resulting from incorrect execution of the measurement process.

# Mathematical Model

**The calculation of probabilities allows the true value to be estimated by a conventionally true value and also enables the assessment of the uncertainty associated with this conventionally true value.**

***For example, a length was measured with a chain 50 times consecutively, and the recorded values are as follows***

| | | |
|---|---|---|
| 2 | mesures à | 74,54 m |
| 3 | mesures à | 74,55 m |
| 5 | mesures à | 74,56 m |
| 8 | mesures à | 74,57 m |
| 10 | mesures à | 74,58 m |
| 9 | mesures à | 74,59 m |
| 6 | mesures à | 74,60 m |
| 4 | mesures à | 74,61 m |
| 2 | mesures à | 74,62 m |
| 1 | mesures à | 74,63 m |




***The resulting distribution curve of these measurements always has the same shape and is called a Gaussian curve.***

# Statistics on Direct Measurements [1]

## Arithmetic Mean and Mean Error

Soit un ensemble de mesures (appelé 'population') : $x_1$ , $x_2$ , $x_3$ , $x_4$ , $x_5$ ,…, $x_n$ .

La moyenne arithmétique notée $x$ est donnée par la formule :

$$x = \frac{x_1 + x_2 + x_3 + \ldots\ldots\ldots \;\; .. + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Les écarts ( $e$ ) à la moyenne arithmétique sont appelés, écarts, erreurs apparentes ou résidus qui sont données par les relations :

$$e_1 = x_1 - x \;\; , \;\; e_2 = x_2 - x \; , \quad e_3 = x_1 - x \; , \quad e_n = x_n - x \; .$$

Leur somme algébrique est nulle. Elle est donnée par la relation :

$$e_1 + e_2 + e_3 + \ldots\ldots + e_n = 0 .$$

# Statistics on Direct Measurements [2]

## Standard Deviations

Appelés aussi erreurs moyennes quadratiques, notée '$emq$', **ou '$\sigma$'** d'une mesure isolée. L'écart type est égal à la racine carrée de la moyenne arithmétique des carrés des écarts à la moyenne. On a pour un grand nombre de mesure :

$$\sigma = \sqrt{\frac{e_1^2 + e_2^2 + e_3^2 + \ldots\ldots + e_n^2}{n}}$$

Pour un nombre limité de mesure, la meilleure estimation est donnée par l'erreur moyenne quadratique.

$$\sigma = \sqrt{\frac{e_1^2 + e_2^2 + e_3^2 + \ldots\ldots + e_n^2}{n-1}}$$

Cette dernière relation définit la précision d'opération de mesures avec plus d'exactitude.

# Statistics on Direct Measurements [3]

## Probable Error

Also called the **equiprobable deviation** of a single measurement, it is the deviation that has a **50% probability of not being exceeded** in absolute value.

$$\varepsilon_p = 0{,}68\ \sigma$$
$$\varepsilon_p \simeq 2/3\ \sigma$$

## Maximum Error or Tolerance

$$\varepsilon_m = 4\ \varepsilon_p \simeq 2{,}66\ \sigma$$

This value defines the **limit beyond which deviations are no longer considered errors** but are most likely **blunders**.

# STATISTIQUES SUR LES MESURES [3]

## Composition of Standard Deviations :

Lorsqu'une mesure est entachée de plusieurs erreurs accidentelles, l'erreur moyenne quadratique **résultante** est donnée par la relation .

$$Emq=\sqrt{emq_1^2+emq_2^2+emq_3^2+\dots\dots\dots+emq_n^2}$$

Lorsque toutes les erreurs de toutes les mesures sont égales:

$$Emq=emq\sqrt{n}$$

## Root Mean Square Error of a Product : $F = X\,.\,Y$

$$Emq_F = \sqrt{Emq_X{}^2.Y^2 + Emq_Y{}^2.X^2}$$

## Root Mean Square Error of a Mean :

L'erreur de la moyenne arithmétique de *n* mesures de la même quantité effectuées avec la même précision $emq$ est donnée par : $Emq_n^{moyenne} = + \frac{emq}{\sqrt{n}}$

# Applications [1]

## Application 1

Soit les dix mesures de distances suivantes:

$D_1 = 120{,}429m$ ; $D_2 = 120{,}448m$ ; $D_3 = 120{,}435m$ ; $D_4 = 120{,}433m$ ; $D_5 = 120{,}441m$ ;
$D_6 = 120{,}424m$ ; $D_7 = 120{,}440m$ ; $D_8 = 120{,}437m$ ; $D_9 = 120{,}434m$ ; $D_{10} = 120{,}439m$ ;

1. Calculer la moyenne arithmétique ;
2. Calculer les erreurs ($d_1$, $d_2$, $d_3$,....., $d_n$) ;
3. Calculer l'erreur moyenne quadratique;
4. Calculer l'erreur probable;
5. Calculer l'erreur maximum.