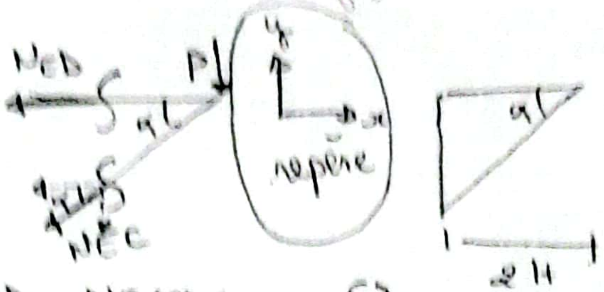
Correction Examen Mec Rat (2025-2026)

Exo: Treillis 01

détermination des efforts au niveau des barres (Tensions)

- nœud E:



$\operatorname{tg}\alpha = \frac{H}{2H} = \frac{1}{2}$

$\alpha = \operatorname{artg}\frac{1}{2}$

$\alpha = 26,56°$

$\begin{cases} \sin\alpha = 0,4472 \\ \cos\alpha = 0,8944 \end{cases}$

$\sum F/x = 0 \Rightarrow -N_{ED} - N_{EC}\cos\alpha = 0$ ... (1)

$\sum F/y = 0 \Rightarrow -P - N_{EC}\sin\alpha = 0 \Rightarrow \boxed{N_{EC} = \frac{-P}{\sin\alpha}}$

$\boxed{N_{EC} = -2,24P}$ ... (3) (compression)

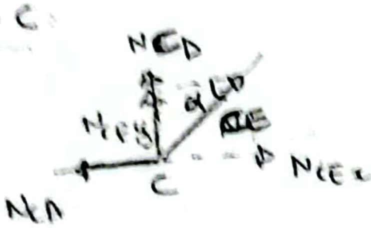
(3) dans (1) $-N_{ED} - N_{EC}\cos\alpha = 0$

$N_{ED} = -N_{EC}\cos\alpha = -\left(\frac{-P}{\sin\alpha}\right)\cos\alpha$

$= \frac{P}{\operatorname{tg}\alpha} = \boxed{2P}$ (Traction)

$\begin{cases} N_{EC} = \frac{-P}{\sin\alpha} = -2,24P \\ N_{ED} = 2P \end{cases}$ 0,75

- nœud C



$\sum F/x = 0 \Rightarrow -N_{CA} + N_{CE}\cos\alpha = 0$ ... (4)

$\sum F/y = 0 \Rightarrow N_{CD} + N_{CE}\sin\alpha$ ... (5)

de (5) $N_{CD} = -N_{CE}\sin\alpha$

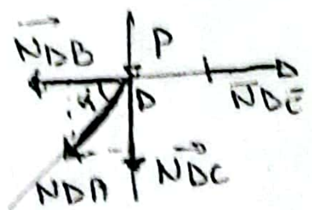
$\boxed{N_{CD}} = -\left(\frac{-P}{\sin\alpha}\right)\sin\alpha = \boxed{P}$ (Traction)

de (4) $-N_{CA} + N_{CE}\cos\alpha = 0$

$\boxed{N_{CA}} = N_{CE}\cos\alpha = -2,24P\cos\alpha = \boxed{-2P}$ (compression)

$\begin{cases} N_{CD} = P \\ N_{CA} = -2P \end{cases}$ 0,75

nœud D:



$\sum F/x = 0 \Rightarrow$

$N_{DE} - N_{DB} - N_{DA}\cos\alpha = 0$ ... (6)

$\sum F/y = 0 \Rightarrow -P - N_{DC} - N_{DA}\sin\alpha = 0$ ... (7)

de (7) $\Rightarrow N_{DA} = \frac{-P - N_{DC}}{\sin\alpha}$

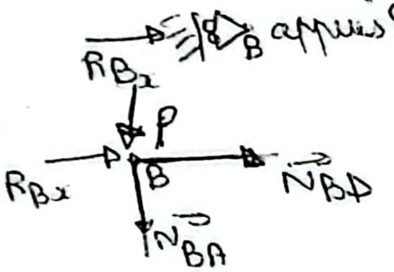
$N_{DA} = \frac{-P-P}{\sin\alpha} = \frac{-2P}{\sin\alpha} = -4,48P$ (compression) de (6) $N_{DE} - N_{DB} - N_{DA}\cos\alpha = 0$

$N_{DB} = N_{DE} - N_{DA}\cos\alpha$

$\boxed{N_{DB} = 2P - (-4{,}48P\cos\alpha)} = \boxed{6P}$ (Traction)

$\begin{cases} N_{DB} = 6P \\ N_{DA} = -4{,}48P \end{cases}$ (0,75)

• Noeud B :



$\Sigma F/x = 0 \Rightarrow$

$R_{Bx} + N_{BD} = 0 \ldots (8)$

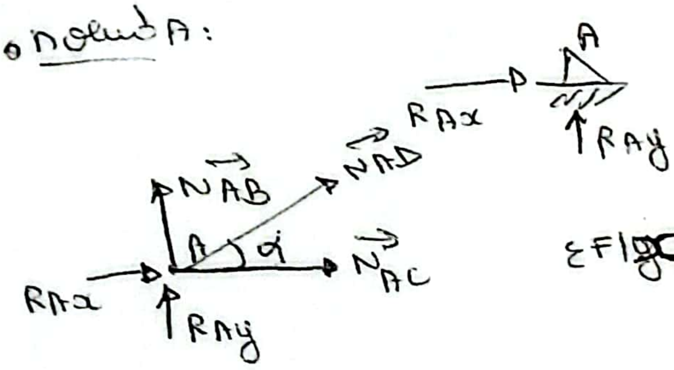
$\Rightarrow \boxed{R_{Bx} = -N_{BD} = -6P}$ (compression)

$\Sigma F/y = 0 \Rightarrow -P - N_{BA} = 0 \ldots (9)$

$\boxed{N_{BA} = -P}$ (compression)

$\begin{cases} R_{Bx} = -6P \\ N_{BA} = -P \end{cases}$ (0,75)

• Noeud A :



$\Sigma F/x = 0 \Rightarrow N_{AC} - R_{Ax} + N_{AD}\cos\alpha = 0$

$R_{Ax} = N_{AD}\cos\alpha - N_{AC}$

$R_{Ax} = -(-4{,}48P)\cos\alpha - (-2P)$

$R_{Ax} = +4P + 2P = 6P$ (Traction)

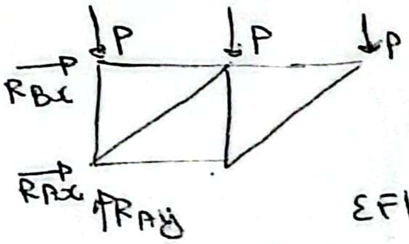
$\Sigma F/y = 0 \Rightarrow R_{Ay} + N_{AB} + N_{AD}\sin\alpha$

$R_{Ay} = -N_{AB} - N_{AD}\sin\alpha$

$R_{Ay} = P + 2P = \boxed{3P}$

$\begin{cases} R_{Ax} = 6P \\ R_{Ay} = 3P \end{cases}$ (0,75)

Verification :



$\Sigma F/x = 0$ $R_{Ax} + R_{Bx} = 0$

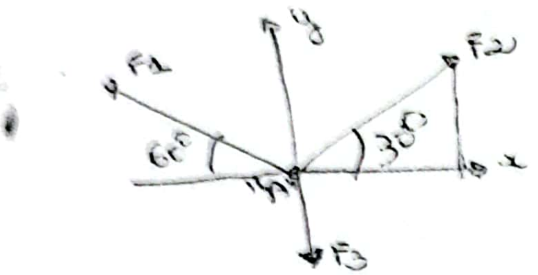
$6P - 6P = 0$ (C.V)

$\Sigma F/y = 0 \Rightarrow R_{Ay} - 3P = 0$

$3P - 3P = 0$ (C.V)

exercice n°3 : (5 point)

$F_3 = P = 9{,}4N$



P.F.S : equilibre du noeud "A"

$\vec{F_1} + \vec{F_2} + \vec{F_3} = \vec{0}$ (1)

$\begin{cases} \Sigma F|_x = 0 \Rightarrow -F_1 \cos 60 + F_2 \cos 30° = 0 \ldots (1) \\ \Sigma F|_y = 0 \Rightarrow F_1 \sin 60° + F_2 \sin 30° = F_3 \ldots (2) \end{cases}$ (1) (1)

(1) $F_1 = \dfrac{F_2 \cos 30°}{\cos 60} = \sqrt{3} F_2 \ldots (3)$

(3) dans (2) $\sqrt{3}F_2 + F_2 \sin 30° = F_3$

$\Rightarrow F_2(\sqrt{3} + \sin 30°) = F_3$

$F_2 = \dfrac{F_3 \ (9{,}4N)}{\sqrt{3} + \sin 30°} = 0{,}2N$

$F_1 = \sqrt{3}F_2 = \sqrt{3}(0{,}2)$ (1)

$\boxed{F_1 = 0{,}346N}$

donc F_1 = poteau = 0,346N

F_2 = poids (1) $\boxed{0{,}2N}$ (1)

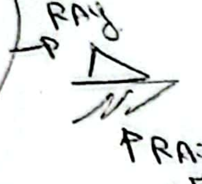
(5)

Questions de cours :

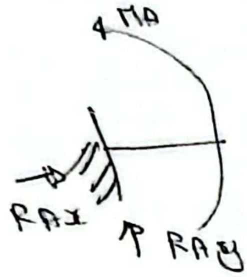
1) Type d'appuis :

appuis simple

une réaction bloquer



deux réactions bloquées (1)



; 3 réaction bloquer.

une articulation sphérique

2) on applique les forces dans un système de treillis "au noeud" (1)

3) Théorème de guldin (voir cours) (1)

element linéique → 1 surface

1 surface → un volume

1) Types d'appuis (voir le cours)

appuis simple - appuis double

encastrement (6) + articulation sphérique + articulation (4 ...

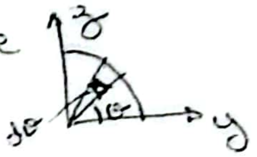
2) donner le torseur au pt "A" et "B"

$$T_A \begin{Bmatrix} R_{Ax} & 0 \\ R_{Ay} & 0 \\ 0 & 0 \end{Bmatrix} \qquad T_A \begin{Bmatrix} 6P & 0 \\ 3P & 0 \\ 0 & 0 \end{Bmatrix} \quad 0,75$$

$$T_B \begin{Bmatrix} R_{Bx} & 0 \\ 0 & 0 \\ 0 & 0 \end{Bmatrix} \qquad T_B \begin{Bmatrix} -6P & 0 \\ 0 & 0 \\ 0 & 0 \end{Bmatrix} \quad 0,75$$

Exercice n°2 : (5 points)

1) le $\frac{1}{4}$ disque



1ère méthode :

$0 \leq \theta \leq \frac{\pi}{2}$

$0 \leq r \leq a$

$dm = \sigma\, ds$; $ds = r\,dr\,d\theta$

$m = \int dm = \int \sigma\, ds = \iint \sigma\, r\,dr\,d\theta = \sigma \int_0^a r\,dr \int_0^{\frac{\pi}{2}} d\theta$ 0,5

$$\boxed{m = \frac{\sigma a^2 \pi}{4}}$$ 0,25

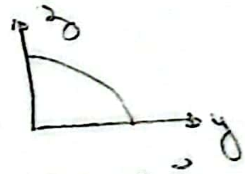
$y_G = \frac{1}{m}\int y\, dm = \frac{4}{\sigma a^2 \pi}\int y\, \sigma\, ds$

$\begin{cases} y = r\cos\theta \\ z = r\sin\theta \end{cases}$ 0,5 $\Rightarrow y_G = \frac{4}{\sigma a^2 \pi}\int r\cos\theta\, r\,dr\,d\theta$ 0,25

0,25 $\frac{4a}{3\pi}$

$z_G = \frac{1}{m}\int z\, dm = \frac{4a}{3\pi}$ 0,25

b) 2ème méthode (théorème de guldin)



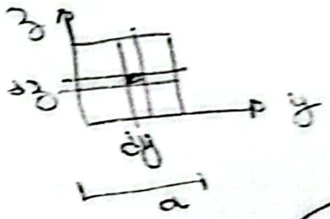
$\begin{cases} V = 2\pi\, y_G . S \\ V = 2\pi\, z_G . S \end{cases}$

$z_G = y_G = \frac{V_{\frac{1}{2}\text{ sphère}}}{2\pi . S_{\frac{1}{4}\text{ disque}}}$; $\begin{cases} V_{\text{sphère}} = \frac{4}{3}\pi a^3 \\ S_{\text{disque}} = \pi a^2 \end{cases}$

0,75

$\begin{cases} y_G = z_G = \frac{4a}{3\pi} \end{cases}$

2) Surface carrée



$0 \leq y \leq a$

$0 \leq z \leq a$

$dm = \sigma\, ds = \sigma\, dy\, dz$ 0,5

$m = \int dm = \int \sigma\, ds = \sigma a^2$ 0,75

$y_G = \frac{1}{m}\int y\, dm = \frac{1}{\sigma a^2}\iint \sigma\, y\, dy\, dz$

$= \frac{\cancel{\sigma}}{\cancel{\sigma} a^2}\int_0^a y\,dy \int_0^a dz = \frac{a}{2}$ 0,25

0,25

$z_G = \frac{1}{m}\int z\, dm = \frac{a}{2}$ 0,25

3) section hachurée :

$y_G = \frac{\sum m_i y_{Gi}}{\sum m_i} = \frac{m_1 y_{G1} - m_2 y_{G2}}{m_1 + m_2}$ 0,25

$y_G = z_G = \frac{a}{2\left(1 - \frac{\pi}{4}\right)}$ 0,5