

Data Envelopment Analysis – Exercises

1. **Primal vs. dual model:** We have set of 4 car garages. Each have 2 inputs (staff number and showroom space) and 2 outputs (number of sold cars and profit). You can find a full version of this example here: http://support.sas.com/documentation/cdl/en/ormpex/65555/HTML/default/viewer.htm#ormpex_ex22_sect001.htm.

	Staff	Showroom space (100m ²)	Sales (1000s)	Profit (millions)
Winchester	7	8	2	1.5
Andover	6	6	2.3	1.6
Basingstoke	2	3	0.8	0.5
Poole	14	9	2.6	1.9

Write a primary and dual CCR DEA (input-oriented) model for **Winchester**.

Primal model

Objective function:

Max
(maximize weighted sum of outputs)

Subject to:

.....
(weighted sum of inputs = 1)

.....

.....

.....

.....
(DMUs' efficiency scores ≤ 1)

.....
(weights non-negative)

Dual model

Objective function:

Min
(minimize θ)

Subject to:

.....

.....
(combination of inputs not greater than analyzed DMU's multiplied by θ)

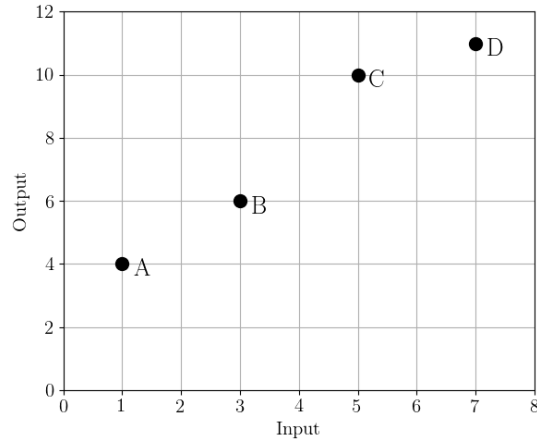
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.....
(combination of output not less than analyzed DMU's.)

.....
(weights non-negative)

2. **CCR vs. BCC:** Draw efficient frontiers for CCR and BCC models for a given example with a single input and a single output. The data is presented in the below table. Then, answer the questions.

	Input	Output
A	1	4
B	3	6
C	5	10
D	7	11

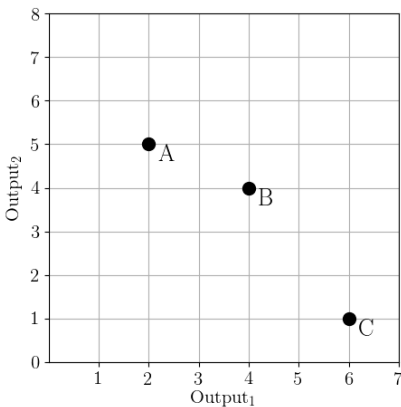


Input oriented CCR: DMU B should decrease its inputs by units to become efficient.

Output oriented CCR: DMU B should increase its outputs by units to become efficient.

3. **Super - efficiency:** compute the super-efficiency value for DMU_B in the example below (2 equal inputs and 2 outputs).

$$SE(B) = \dots$$



4. **Efficiency distribution:** Using a Monte Carlo method we may estimate efficiency distribution for some DMU. We generated 10 weight vectors. Efficiency scores of analyzed DMU for these samples are given in the table below. We decided to analyze distribution in 4 buckets with equal width. Estimate EAIIs for this DMU.

Sample no	1	2	3	4	5	6	7	8	9	10
Efficiency score	0.78	0.2	0.01	0.97	1	0.14	0.37	0.63	0.53	0.26

Bucket	[0.0, 0.25)	[0.25, 0.5)	[0.5, 0.75)	[0.75, 1]
Samples count				
Estimated EAI				